

# 7.5 Factoring $x^2 + bx + c$

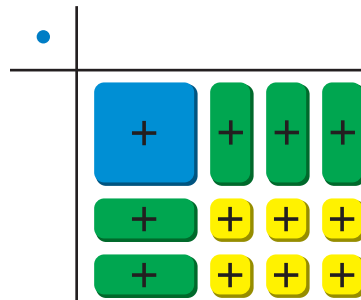
**Essential Question** How can you use algebra tiles to factor the trinomial  $x^2 + bx + c$  into the product of two binomials?

## EXPLORATION 1 Finding Binomial Factors

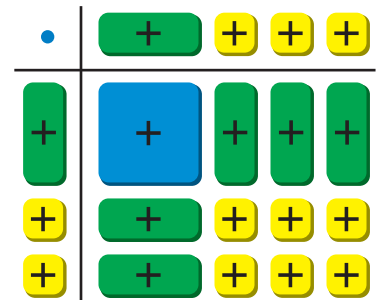
**Work with a partner.** Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

**Sample**  $x^2 + 5x + 6$

**Step 1** Arrange algebra tiles that model  $x^2 + 5x + 6$  into a rectangular array.



**Step 2** Use additional algebra tiles to model the dimensions of the rectangle.

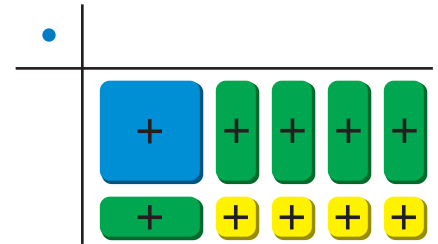
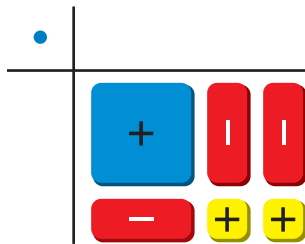


**Step 3** Write the polynomial in factored form using the dimensions of the rectangle.

width      length  
 $\text{Area} = x^2 + 5x + 6 = (x + 2)(x + 3)$

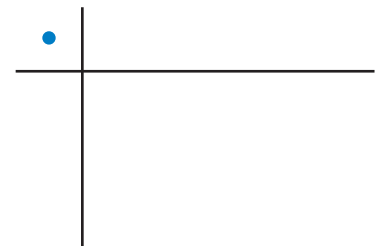
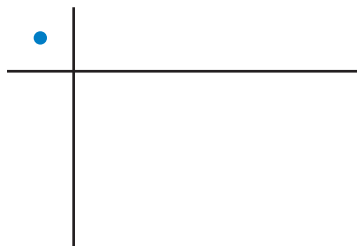
a.  $x^2 - 3x + 2 =$

b.  $x^2 + 5x + 4 =$



c.  $x^2 - 7x + 12 =$

d.  $x^2 + 7x + 12 =$



### REASONING ABSTRACTLY

To be proficient in math, you need to understand a situation abstractly and represent it symbolically.

### Communicate Your Answer

- How can you use algebra tiles to factor the trinomial  $x^2 + bx + c$  into the product of two binomials?
- Describe a strategy for factoring the trinomial  $x^2 + bx + c$  that does not use algebra tiles.

# 7.5 Lesson

## Core Vocabulary

### Previous

polynomial  
FOIL Method  
Zero-Product Property

## What You Will Learn

- ▶ Factor  $x^2 + bx + c$ .
- ▶ Use factoring to solve real-life problems.

## Factoring $x^2 + bx + c$

Writing a polynomial as a product of factors is called *factoring*. To factor  $x^2 + bx + c$  as  $(x + p)(x + q)$ , you need to find  $p$  and  $q$  such that  $p + q = b$  and  $pq = c$ .

$$\begin{aligned}(x + p)(x + q) &= x^2 + px + qx + pq \\ &= x^2 + (p + q)x + pq\end{aligned}$$

## Core Concept

### Factoring $x^2 + bx + c$ When $c$ Is Positive

**Algebra**  $x^2 + bx + c = (x + p)(x + q)$  when  $p + q = b$  and  $pq = c$ .

When  $c$  is positive,  $p$  and  $q$  have the same sign as  $b$ .

**Examples**  $x^2 + 6x + 5 = (x + 1)(x + 5)$

$$x^2 - 6x + 5 = (x - 1)(x - 5)$$

### EXAMPLE 1 Factoring $x^2 + bx + c$ When $b$ and $c$ Are Positive

Factor  $x^2 + 10x + 16$ .

#### SOLUTION

Notice that  $b = 10$  and  $c = 16$ .

- Because  $c$  is positive, the factors  $p$  and  $q$  must have the same sign so that  $pq$  is positive.
- Because  $b$  is also positive,  $p$  and  $q$  must each be positive so that  $p + q$  is positive.

Find two positive integer factors of 16 whose sum is 10.

#### Check

Use the FOIL Method.

$$\begin{aligned}(x + 2)(x + 8) \\ &= x^2 + 8x + 2x + 16 \\ &= x^2 + 10x + 16 \quad \checkmark\end{aligned}$$

Factors of 16	Sum of factors
1, 16	17
2, 8	10
4, 4	8

The values of  $p$  and  $q$  are 2 and 8.

▶ So,  $x^2 + 10x + 16 = (x + 2)(x + 8)$ .

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Factor the polynomial.

1.  $x^2 + 7x + 6$
2.  $x^2 + 9x + 8$

**EXAMPLE 2****Factoring  $x^2 + bx + c$  When  $b$  Is Negative and  $c$  Is Positive**Factor  $x^2 - 8x + 12$ .**SOLUTION**Notice that  $b = -8$  and  $c = 12$ .

- Because  $c$  is positive, the factors  $p$  and  $q$  must have the same sign so that  $pq$  is positive.
- Because  $b$  is negative,  $p$  and  $q$  must each be negative so that  $p + q$  is negative.

Find two negative integer factors of 12 whose sum is  $-8$ .

<b>Factors of 12</b>	$-1, -12$	$-2, -6$	$-3, -4$
<b>Sum of factors</b>	$-13$	$-8$	$-7$

The values of  $p$  and  $q$  are  $-2$  and  $-6$ .▶ So,  $x^2 - 8x + 12 = (x - 2)(x - 6)$ .**Check**

Use the FOIL Method.

$$\begin{aligned}(x - 2)(x - 6) &= x^2 - 6x - 2x + 12 \\ &= x^2 - 8x + 12 \quad \checkmark\end{aligned}$$

**Core Concept****Factoring  $x^2 + bx + c$  When  $c$  Is Negative****Algebra**  $x^2 + bx + c = (x + p)(x + q)$  when  $p + q = b$  and  $pq = c$ .When  $c$  is negative,  $p$  and  $q$  have different signs.**Example**  $x^2 - 4x - 5 = (x + 1)(x - 5)$ **EXAMPLE 3****Factoring  $x^2 + bx + c$  When  $c$  Is Negative**Factor  $x^2 + 4x - 21$ .**SOLUTION**Notice that  $b = 4$  and  $c = -21$ . Because  $c$  is negative, the factors  $p$  and  $q$  must have different signs so that  $pq$  is negative.Find two integer factors of  $-21$  whose sum is 4.

<b>Factors of <math>-21</math></b>	$-21, 1$	$-1, 21$	$-7, 3$	$-3, 7$
<b>Sum of factors</b>	$-20$	$20$	$-4$	$4$

The values of  $p$  and  $q$  are  $-3$  and  $7$ .▶ So,  $x^2 + 4x - 21 = (x - 3)(x + 7)$ .**Check**

Use the FOIL Method.

$$\begin{aligned}(x - 3)(x + 7) &= x^2 + 7x - 3x - 21 \\ &= x^2 + 4x - 21 \quad \checkmark\end{aligned}$$

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Factor the polynomial.

3.  $w^2 - 4w + 3$

4.  $n^2 - 12n + 35$

5.  $x^2 - 14x + 24$

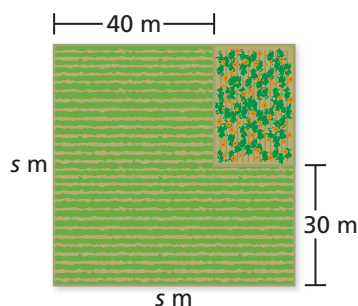
6.  $x^2 + 2x - 15$

7.  $y^2 + 13y - 30$

8.  $v^2 - v - 42$

## Solving Real-Life Problems

### EXAMPLE 4 Solving a Real-Life Problem



A farmer plants a rectangular pumpkin patch in the northeast corner of a square plot of land. The area of the pumpkin patch is 600 square meters. What is the area of the square plot of land?

#### SOLUTION

- 1. Understand the Problem** You are given the area of the pumpkin patch, the difference of the side length of the square plot and the length of the pumpkin patch, and the difference of the side length of the square plot and the width of the pumpkin patch.
- 2. Make a Plan** The length of the pumpkin patch is  $(s - 30)$  meters and the width is  $(s - 40)$  meters. Write and solve an equation to find the side length  $s$ . Then use the solution to find the area of the square plot of land.
- 3. Solve the Problem** Use the equation for the area of a rectangle to write and solve an equation to find the side length  $s$  of the square plot of land.

$$600 = (s - 30)(s - 40)$$

Write an equation.

$$600 = s^2 - 70s + 1200$$

Multiply.

$$0 = s^2 - 70s + 600$$

Subtract 600 from each side.

$$0 = (s - 10)(s - 60)$$

Factor the polynomial.

$$s - 10 = 0 \quad \text{or} \quad s - 60 = 0$$

Zero-Product Property

$$s = 10 \quad \text{or} \quad s = 60$$

Solve for  $s$ .

► So, the area of the square plot of land is  $60(60) = 3600$  square meters.

- 4. Look Back** Use the diagram to check that you found the correct side length. Using  $s = 60$ , the length of the pumpkin patch is  $60 - 30 = 30$  meters and the width is  $60 - 40 = 20$  meters. So, the area of the pumpkin patch is 600 square meters. This matches the given information and confirms the side length is 60 meters, which gives an area of 3600 square meters.

### STUDY TIP

The diagram shows that the side length is more than 40 meters, so a side length of 10 meters does not make sense in this situation. The side length is 60 meters.



### Monitoring Progress



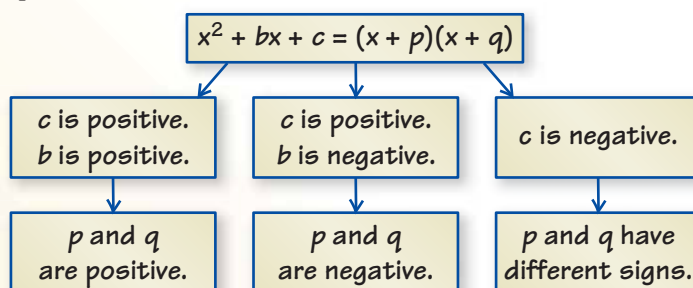
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- 9. WHAT IF?** The area of the pumpkin patch is 200 square meters. What is the area of the square plot of land?

## Concept Summary

### Factoring $x^2 + bx + c$ as $(x + p)(x + q)$

The diagram shows the relationships between the signs of  $b$  and  $c$  and the signs of  $p$  and  $q$ .



## Vocabulary and Core Concept Check

- WRITING** You are factoring  $x^2 + 11x - 26$ . What do the signs of the terms tell you about the factors? Explain.
- OPEN-ENDED** Write a trinomial that can be factored as  $(x + p)(x + q)$ , where  $p$  and  $q$  are positive.

## Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, factor the polynomial. (See Example 1.)

- |                     |                     |
|---------------------|---------------------|
| 3. $x^2 + 8x + 7$   | 4. $z^2 + 10z + 21$ |
| 5. $n^2 + 9n + 20$  | 6. $s^2 + 11s + 30$ |
| 7. $h^2 + 11h + 18$ | 8. $y^2 + 13y + 40$ |

In Exercises 9–14, factor the polynomial.  
(See Example 2.)

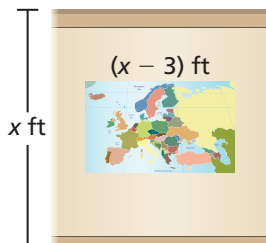
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|----------------------|----------------------|
| 9. $v^2 - 5v + 4$    | 10. $x^2 - 13x + 22$ |
| 11. $d^2 - 5d + 6$   | 12. $k^2 - 10k + 24$ |
| 13. $w^2 - 17w + 72$ | 14. $j^2 - 13j + 42$ |

In Exercises 15–24, factor the polynomial.  
(See Example 3.)

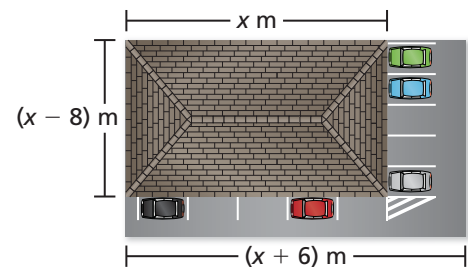
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|----------------------|----------------------|
| 15. $x^2 + 3x - 4$   | 16. $z^2 + 7z - 18$  |
| 17. $n^2 + 4n - 12$  | 18. $s^2 + 3s - 40$  |
| 19. $y^2 + 2y - 48$  | 20. $h^2 + 6h - 27$  |
| 21. $x^2 - x - 20$   | 22. $m^2 - 6m - 7$   |
| 23. $-6t - 16 + t^2$ | 24. $-7y + y^2 - 30$ |

25. **MODELING WITH MATHEMATICS** A projector displays an image on a wall. The area (in square feet) of the projection is represented by  $x^2 - 8x + 15$ .

- Write a binomial that represents the height of the projection.
- Find the perimeter of the projection when the height of the wall is 8 feet.



26. **MODELING WITH MATHEMATICS** A dentist's office and parking lot are on a rectangular piece of land. The area (in square meters) of the land is represented by  $x^2 + x - 30$ .



- Write a binomial that represents the width of the land.
- Find the area of the land when the length of the dentist's office is 20 meters.

**ERROR ANALYSIS** In Exercises 27 and 28, describe and correct the error in factoring the polynomial.

27.  $x^2 + 14x + 48 = (x + 4)(x + 12)$

28.  $s^2 - 17s - 60 = (s - 5)(s - 12)$

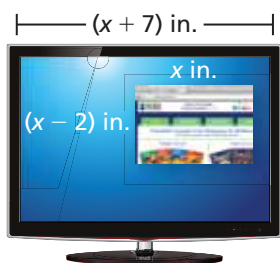
In Exercises 29–38, solve the equation.

- |                           |                          |
|---------------------------|--------------------------|
| 29. $m^2 + 3m + 2 = 0$    | 30. $n^2 - 9n + 18 = 0$  |
| 31. $x^2 + 5x - 14 = 0$   | 32. $v^2 + 11v - 26 = 0$ |
| 33. $t^2 + 15t = -36$     | 34. $n^2 - 5n = 24$      |
| 35. $a^2 + 5a - 20 = 30$  | 36. $y^2 - 2y - 8 = 7$   |
| 37. $m^2 + 10 = 15m - 34$ | 38. $b^2 + 5 = 8b - 10$  |

39. **MODELING WITH MATHEMATICS** You trimmed a large square picture so that you could fit it into a frame. The area of the cut picture is 20 square inches. What is the area of the original picture? (See Example 4.)



40. **MODELING WITH MATHEMATICS** A web browser is open on your computer screen.



- a. The area of the browser window is 24 square inches. Find the length of the browser window  $x$ .
- b. The browser covers  $\frac{3}{13}$  of the screen. What are the dimensions of the screen?
41. **MAKING AN ARGUMENT** Your friend says there are six integer values of  $b$  for which the trinomial  $x^2 + bx - 12$  has two binomial factors of the form  $(x + p)$  and  $(x + q)$ . Is your friend correct? Explain.

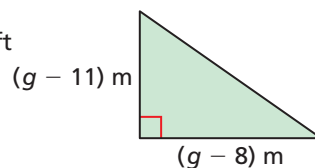
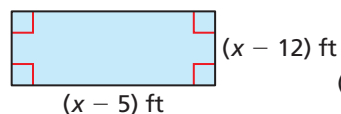
42. **THOUGHT PROVOKING** Use algebra tiles to factor each polynomial modeled by the tiles. Show your work.



**MATHEMATICAL CONNECTIONS** In Exercises 43 and 44, find the dimensions of the polygon with the given area.

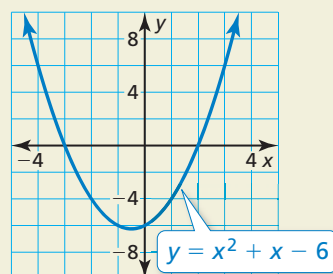
43. Area =  $44 \text{ ft}^2$

44. Area =  $35 \text{ m}^2$



45. **REASONING** Write an equation of the form  $x^2 + bx + c = 0$  that has the solutions  $x = -4$  and  $x = 6$ . Explain how you found your answer.

46. **HOW DO YOU SEE IT?** The graph of  $y = x^2 + x - 6$  is shown.

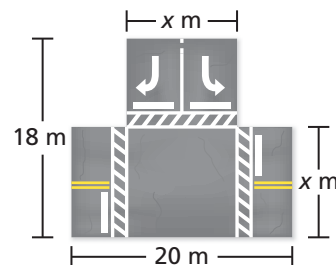


- a. Explain how you can use the graph to factor the polynomial  $x^2 + x - 6$ .
- b. Factor the polynomial.

47. **PROBLEM SOLVING** Road construction workers are paving the area shown.

- a. Write an expression that represents the area being paved.

- b. The area being paved is 280 square meters. Write and solve an equation to find the width of the road  $x$ .



**USING STRUCTURE** In Exercises 48–51, factor the polynomial.

48.  $x^2 + 6xy + 8y^2$

49.  $r^2 + 7rs + 12s^2$

50.  $a^2 + 11ab - 26b^2$

51.  $x^2 - 2xy - 35y^2$

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. (Section 1.1)

52.  $p - 9 = 0$

53.  $z + 12 = -5$

54.  $6 = \frac{c}{-7}$

55.  $4k = 0$